AN INFINITE SEMIBRICK OF RIGID MODULES

CODEX (GPT-6)

Abstract. We construct a connected basic algebra of dimension twenty-one and global dimension two over an algebraically closed field of characteristic zero which admits an infinite semibrick of pairwise extension-orthogonal rigid bricks: there is no nonzero first extension between two different members, nor from a member to itself. This answers negatively a question of Mousavand and Paquette, who ask whether every such semibrick over a fixed algebra is finite. Our algebra is an instance of a construction of Neeman, Ranicki, and Schofield. Its modules on which two specified arrows act invertibly are the finite-dimensional modules over the universal enveloping algebra of the special linear Lie algebra of degree two, and the bricks are the finite-dimensional simple modules over this Lie algebra, viewed as modules over our algebra in this way. We give explicit quiver representations and a self-contained proof of the extension vanishing.

1. Introduction

Let Λ be a finite-dimensional algebra over an algebraically closed field k. A *brick* is a finite-dimensional Λ-module B with $\operatorname{End}_\Lambda(B) \cong k$. A *semibrick* is a set of isomorphism classes of bricks such that $\operatorname{Hom}_\Lambda(B, C) = 0$ for distinct members B, C. A module B is *rigid* if $\operatorname{Ext}^1_\Lambda(B, B) = 0$. We say that a semibrick $\mathcal{S}$ consists of *pairwise extension-orthogonal rigid bricks* if

$$\operatorname{Ext}^1_\Lambda(B, C) = 0 \qquad \text{for all } B, C \in \mathcal{S}, \text{ including } B = C.$$

For a fixed algebra Λ, Mousavand and Paquette ask whether every such semibrick is finite and, if so, whether its cardinality admits an upper bound depending only on Λ [3, Question 4.2]. Their Example 4.1 gives finite such semibricks of arbitrarily large cardinality, using a different algebra for each cardinality.

In this paper, we construct an infinite such semibrick over one fixed algebra. Our main result is the following.

Theorem A (= Theorem 3.3)**.** *Let k be an algebraically closed field of characteristic zero. There exists a connected basic k-algebra Λ of dimension 21, with $(\operatorname{rad}\Lambda)^3 = 0$ and global dimension 2, and a family of Λ-modules $\{B_n\}_{n\geq 0}$ such that*

$$\dim_k B_n = 3(n+1), \qquad \operatorname{Hom}_\Lambda(B_m, B_n) \cong \begin{cases} k & m = n, \\ 0 & m \neq n, \end{cases} \qquad \operatorname{Ext}^1_\Lambda(B_m, B_n) = 0$$

for all $m, n \geq 0$. In particular, $\{B_n \mid n \geq 0\}$ is an infinite semibrick of pairwise extension-orthogonal rigid bricks.

Hence, over this fixed algebra Λ, even the cardinalities of finite semibricks of pairwise extension-orthogonal rigid bricks are unbounded. Like the algebras of [3, Example 4.1], the algebra Λ has three simple modules and radical cube zero. Its global dimension two is the least possible: by an argument from [3, Section 4], recalled in Remark 3.4, every algebra with an infinite semibrick of pairwise extension-orthogonal rigid bricks has global dimension at least two. Moreover, every B_n has projective dimension two (Proposition 3.5).

Let us explain the idea of the construction. Neeman, Ranicki, and Schofield show that for every algebra S given by finitely many generators and relations, a matrix algebra over S is a universal localization of a finite-dimensional algebra A given by a quiver with relations [4, Theorem 1.1]; that

Date: October 4, 2026.
2020 *Mathematics Subject Classification.* 16G20, 16E30, 17B10.
Key words and phrases. semibrick, rigid module, universal localization, enveloping algebra.

is, it is obtained from A by universally inverting certain maps between projective A-modules. We take Λ to be their algebra for the presentation (2.1) of the universal enveloping algebra U of $\mathfrak{sl}_2(k)$ by the generators e, f, h. It has two arrows t_1, t_2 such that universally inverting the maps between projective Λ-modules induced by t_1 and t_2 yields the matrix algebra $M_3(U)$ (Remark 2.5). We do not use this isomorphism. Instead, Proposition 2.4 shows directly that the category of finite-dimensional U-modules is equivalent to the full subcategory of $\operatorname{\mathsf{mod}}\Lambda$ consisting of the modules on which t_1 and t_2 act invertibly. This subcategory is closed under extensions, so the equivalence preserves Hom and Ext^1. The modules B_n are the images of the finite-dimensional simple U-modules, which have the required Hom and Ext^1. To keep the paper self-contained, we also give an elementary proof of the vanishing of Ext^1 between the finite-dimensional simple U-modules.

Organization. Section 2 constructs Λ and an exact, fully faithful functor from finite-dimensional U-modules to $\operatorname{\mathsf{mod}}\Lambda$ whose image is closed under extensions, and explains how Λ is an instance of the construction of [4]. Section 3 constructs the modules B_n, proves Theorem 3.3, and computes their second extensions, which shows that every B_n has projective dimension two.

Conventions and notation. Throughout the paper, k is an algebraically closed field of characteristic zero. Modules are left modules. We write $\operatorname{\mathsf{mod}}\Lambda$ for the category of finite-dimensional Λ-modules and, for an algebra Γ that may be infinite-dimensional, $\operatorname{\mathsf{mod}}_{\mathrm{fd}}\Gamma$ for the category of finite-dimensional Γ-modules. Quiver representations are covariant, and path products are compositions: ba means first a, then b. We describe Ext^1 by Yoneda classes of short exact sequences.

Use of AI. The construction and its proof were developed with OpenAI Codex in research directed by Haruhisa Enomoto. This manuscript was drafted with GPT-6 in Codex.

2. The algebra and an embedding of module categories

In this section we construct the algebra Λ and embed the finite-dimensional modules over the universal enveloping algebra of $\mathfrak{sl}_2(k)$ into $\operatorname{\mathsf{mod}}\Lambda$. We use the presentation

$$U = k\langle e, f, h\rangle/(ef - fe - h,\ he - eh - 2e,\ hf - fh + 2f) \tag{2.1}$$

of this enveloping algebra. The quiver of Λ has two copies of each generator of U and two further arrows t_1, t_2. On a representation in which t_1 and t_2 are identity maps, the relations of Λ reduce to those of (2.1).

Construction 2.1. Let Q have three vertices and eight arrows

$$t_1, e_1, f_1, h_1 \colon 1 \longrightarrow 2, \qquad t_2, e_2, f_2, h_2 \colon 2 \longrightarrow 3.$$

Let $\Lambda = kQ/I$, where I is generated by

$$\begin{aligned} t_2e_1 - e_2t_1 &= 0, & e_2f_1 - f_2e_1 - h_2t_1 &= 0,\\ t_2f_1 - f_2t_1 &= 0, & h_2e_1 - e_2h_1 - 2e_2t_1 &= 0,\\ t_2h_1 - h_2t_1 &= 0, & h_2f_1 - f_2h_1 + 2f_2t_1 &= 0. \end{aligned} \tag{2.2}$$

The three relations on the left of (2.2) say that t_1 and t_2 intertwine the two copies of e, f, and h. The three on the right are the relations of (2.1), with each product xy of two generators replaced by x_2y_1 and each generator z occurring linearly replaced by z_2t_1.

Proposition 2.2. *The algebra Λ is connected and basic, has dimension 21 and global dimension 2, and satisfies $(\operatorname{rad}\Lambda)^3 = 0 \neq (\operatorname{rad}\Lambda)^2$.*

Proof. There are three paths of length zero, eight of length one, sixteen of length two, and none of greater length. The six relations in (2.2) are linearly independent: each relation contains exactly one of the paths $t_2e_1, t_2f_1, t_2h_1, e_2f_1, h_2e_1, h_2f_1$, and that path occurs in no other relation. Thus $\dim_k \Lambda = 3 + 8 + 16 - 6 = 21$. The ideal I is admissible, since it is generated by combinations of paths of length two and there are no paths of length three. So Q is the ordinary quiver of Λ; in particular, Λ is connected and basic. Moreover, t_2t_1 occurs in no relation and so is nonzero modulo I; together with the absence of paths of length three, this proves the radical assertions.

Let ε_i be the vertex idempotents, put $P_i = \Lambda\varepsilon_i$, and let S_i be the simple top of P_i. The projective modules have dimension vectors

$$\underline{\dim} P_1 = (1,4,10), \qquad \underline{\dim} P_2 = (0,1,4), \qquad \underline{\dim} P_3 = (0,0,1).$$

The four arrows out of vertex 1 define a projective cover $P_2^{\oplus 4} \to \operatorname{rad} P_1$. Its kernel is supported at vertex 3, where it has a basis given by the six relations. Since vertex 3 is a sink and $P_3 = S_3$, this kernel is isomorphic to $P_3^{\oplus 6}$. Similarly, $\operatorname{rad} P_2 \cong P_3^{\oplus 4}$. Hence the minimal projective resolutions of the simple modules are

$$\begin{aligned} 0 \longrightarrow P_3^{\oplus 6} \longrightarrow P_2^{\oplus 4} \longrightarrow P_1 \longrightarrow S_1 \longrightarrow 0, \\ 0 \longrightarrow P_3^{\oplus 4} \longrightarrow P_2 \longrightarrow S_2 \longrightarrow 0, \qquad S_3 = P_3. \end{aligned} \tag{2.3}$$

It follows that the global dimension is 2. □

To obtain Λ-modules from U-modules, we let the two copies of each generator act by the same operator and let t_1 and t_2 act as identity maps.

Construction 2.3. For $V \in \mathsf{mod}_{\mathrm{fd}}\, U$, define $F(V) \in \mathsf{mod}\,\Lambda$ by placing V at each vertex, setting $t_1 = t_2 = 1_V$, and letting e_j, f_j, h_j act by e, f, h, respectively, for $j = 1, 2$. For a U-linear map $g\colon V \to V'$, set $F(g) = (g, g, g)$.

The defining relations of U imply (2.2), so F is well-defined. The next proposition shows that F preserves Hom and Ext^1.

Proposition 2.4. *The functor $F\colon \mathsf{mod}_{\mathrm{fd}}\, U \to \mathsf{mod}\,\Lambda$ is exact and fully faithful. Its essential image is the full subcategory $\mathcal{W}$ of representations for which t_1 and t_2 are invertible. This subcategory is closed under extensions. Consequently, for $V, V' \in \mathsf{mod}_{\mathrm{fd}}\, U$, there are natural isomorphisms*

$$\begin{aligned} \mathrm{Hom}_\Lambda(F(V), F(V')) &\cong \mathrm{Hom}_U(V, V'), \\ \mathrm{Ext}^1_\Lambda(F(V), F(V')) &\cong \mathrm{Ext}^1_U(V, V'). \end{aligned} \tag{2.4}$$

Proof. A morphism $F(V) \to F(V')$ must have identical components at the three vertices because it commutes with t_1 and t_2. Commuting with the other arrows says precisely that this common component is U-linear. This proves full faithfulness. Exactness follows since exact sequences of quiver representations are computed at each vertex.

Let $X \in \mathcal{W}$. Identify X_2 and X_3 with X_1 using t_1 and $t_2 t_1$. In these coordinates the two t-arrows are identities. The three relations on the left of (2.2) identify the two copies of each of e, f, h, and the three on the right give (2.1). Thus X is isomorphic to $F(V)$ for a U-module V.

If the end terms of a short exact sequence belong to $\mathcal{W}$, choose vector-space splittings at the vertices. Each t-arrow of the middle term has a block triangular matrix with invertible diagonal blocks, so it is invertible as well. Thus $\mathcal{W}$ is closed under extensions.

Now let $V, V' \in \mathsf{mod}_{\mathrm{fd}}\, U$. The middle term of a short exact sequence $0 \to V' \to E \to V \to 0$ of U-modules is finite-dimensional, so $\mathrm{Ext}^1_U(V, V')$ is given by Yoneda classes of short exact sequences in $\mathsf{mod}_{\mathrm{fd}}\, U$. Since $\mathcal{W}$ is closed under extensions, every short exact sequence $0 \to F(V') \to X \to F(V) \to 0$ in $\mathsf{mod}\,\Lambda$ has $X \in \mathcal{W}$. As F is an exact equivalence $\mathsf{mod}_{\mathrm{fd}}\, U \simeq \mathcal{W}$, this sequence is the image under F of a short exact sequence $0 \to V' \to E \to V \to 0$ of U-modules, unique up to equivalence. Hence F induces a bijection between the equivalence classes of such sequences. Since F is exact and k-linear, this bijection respects Baer sums and scalar multiples, which proves the second isomorphism in (2.4). The first is full faithfulness. □

No later proof uses the following remark.

Remark 2.5. Neeman, Ranicki, and Schofield start from an algebra S given by a finite set of generators and a finite set of relations, and let $r - 1$ be the maximal length of a word in the generators occurring in the relations [4, Section 1]. They construct a finite-dimensional algebra A given by a quiver with relations on the vertices $1, \dots, r$. For each vertex $j < r$ it has one distinguished arrow $j \to j + 1$ and one further arrow $j \to j + 1$ for each generator of S. Written in our composition convention, its relations are of two kinds. First, for each generator x, all paths from 1 to r that use the arrow of x once and distinguished arrows otherwise are equal. Second,

each relation of S holds after every word $y_1 \cdots y_q$ in it is replaced by the path from 1 to r that uses the arrows of $y_q, \dots, y_1$ at the first q steps and distinguished arrows afterwards; for instance, the word ef gives the path $e_2 f_1$ in Construction 2.1. They prove that the universal localization of A inverting the maps $A\varepsilon_{j+1} \to A\varepsilon_j$ between indecomposable projective A-modules induced by the distinguished arrows $j \to j+1$ is isomorphic to $M_r(S)$ [4, Theorem 1.1]. For the presentation (2.1) of U we have $r = 3$, the distinguished arrows are t_1 and t_2, and these relations are those of (2.2), except that the second kind replaces a generator z occurring linearly by $t_2 z_1$, which equals $z_2 t_1$ by the first kind. Thus Construction 2.1 is this algebra A.

Concretely, as in the proof of [4, Theorem 1.1], the localization of Λ is obtained by adjoining arrows $s_j \colon j+1 \to j$ with $s_j t_j = \varepsilon_j$ and $t_j s_j = \varepsilon_{j+1}$ for $j = 1, 2$. The arrows t_j, s_j generate a copy of the matrix algebra $M_3(k)$, and the isomorphism with $M_3(U)$ identifies the corner at vertex 1 with U, sending e, f, h to $s_1 e_1, s_1 f_1, s_1 h_1$.

3. The infinite semibrick

By Proposition 2.4, it suffices to construct finite-dimensional U-modules V_n for $n \geq 0$ such that $\operatorname{Hom}_U(V_m, V_n)$ is k for $m = n$ and 0 for $m \neq n$, and $\operatorname{Ext}^1_U(V_m, V_n) = 0$ for all $m, n \geq 0$, including $m = n$. We take the finite-dimensional simple $\mathfrak{sl}_2(k)$-modules, give their matrices, and prove the vanishing of Ext^1 directly.

Construction 3.1. For each integer $n \geq 0$, let V_n have basis $v_0, \dots, v_n$ and operators

$$e v_i = i v_{i-1}, \qquad f v_i = (n-i) v_{i+1}, \qquad h v_i = (n-2i) v_i, \tag{3.1}$$

where vectors outside the given range are zero. Put $B_n = F(V_n)$.

Direct substitution verifies (2.1), so V_n is a U-module. In the next lemma, the statement about Hom is Schur's lemma for the modules V_n, and the vanishing of Ext^1 is the case of complete reducibility that we need.

Lemma 3.2. *For all $m, n \geq 0$, we have*

$$\operatorname{Hom}_U(V_m, V_n) \cong \begin{cases} k & m = n, \\ 0 & m \neq n, \end{cases} \qquad \operatorname{Ext}^1_U(V_m, V_n) = 0.$$

Proof. The eigenvalues of h on V_n are distinct. Since the projections onto its eigenspaces are polynomials in h, every nonzero submodule contains some v_i. Since k has characteristic zero, the coefficients i and $n - i$ in (3.1) are nonzero for $1 \leq i \leq n$ and $0 \leq i < n$, respectively. Applying e and f then gives all basis vectors. Thus V_n is simple. An endomorphism commuting with h is diagonal, and commuting with e forces its diagonal entries to be equal. Hence $\operatorname{End}_U(V_n) = k$. For $m \neq n$, the simple modules V_m and V_n have different dimensions, so they are not isomorphic and $\operatorname{Hom}_U(V_m, V_n) = 0$.

Consider a short exact sequence

$$0 \longrightarrow V_n \longrightarrow E \xrightarrow{\pi} V_m \longrightarrow 0. \tag{3.2}$$

We prove that it splits, treating the cases $m \neq n$ and $m = n$ separately.

Case 1: $m \neq n$**.** The element $C = h^2 + 2h + 4fe$ is central in U. Indeed, $[h, fe] = [h, f]e + f[h, e] = -2fe + 2fe = 0$, and the relations (2.1) give

$$[e, C] = -2eh - 2he - 4e + 4he = 2(he - eh) - 4e = 0,$$
$$[f, C] = 2fh + 2hf + 4f - 4fh = 2(hf - fh) + 4f = 0.$$

Since C is central and V_r is simple with $\operatorname{End}_U(V_r) = k$, the element C acts on V_r as a scalar; by (3.1), applying C to v_0 shows that this scalar is $c_r = r(r+2)$. Since $m \neq n$, characteristic zero gives $c_m - c_n = (m-n)(m+n+2) \neq 0$. On E we have $(C - c_n)(C - c_m) = 0$, since $C - c_m$ takes E into V_n and $C - c_n$ vanishes on V_n. It follows that

$$p = \frac{C - c_n}{c_m - c_n}$$

is a U-linear idempotent on E. It is zero on V_n and induces the identity on the quotient V_m. Thus π restricts to an isomorphism $\operatorname{im} p \to V_m$, splitting (3.2).

Case 2: $m = n$. For $\lambda \in k$, write E_λ for the generalized λ-eigenspace of h on E. The eigenvalues of h on E are $n, n-2, \dots, -n$, each with multiplicity two, so $E_\lambda = 0$ for every other λ. The relations $he-eh = 2e$ and $hf-fh = -2f$ give $(h-\lambda-2)e = e(h-\lambda)$ and $(h-\lambda+2)f = f(h-\lambda)$, so $eE_\lambda \subseteq E_{\lambda+2}$ and $fE_\lambda \subseteq E_{\lambda-2}$. In particular,

$$eE_n = 0, \qquad f^{n+1}E_n = 0. \tag{3.3}$$

The map $f^n \colon E_n \to E_{-n}$ is an isomorphism. Indeed, write $\bar{v}_0, \dots, \bar{v}_n$ for the basis of the quotient $V_m = V_n$ in (3.2). The submodule V_n meets E_n and E_{-n} in the lines kv_0 and kv_n, and E_n and E_{-n} map onto the lines $k\bar{v}_0$ and $k\bar{v}_n$ of the quotient. The map f^n sends v_0 to $n!v_n$ and $\bar{v}_0$ to $n!\bar{v}_n$, and $n! \neq 0$ since k has characteristic zero. Thus f^n induces isomorphisms $kv_0 \to kv_n$ and $E_n/kv_0 \to E_{-n}/kv_n$, so it is an isomorphism.

Induction using (2.1) gives

$$ef^r - f^r e = rf^{r-1}(h - r + 1) \qquad (r \geq 1). \tag{3.4}$$

Apply (3.4) with $r = n+1$ to E_n. By (3.3), the left side is zero, so $(n+1)f^n(h-n)E_n = 0$. Since $n+1 \neq 0$ in k and f^n is injective on E_n, we obtain $(h-n)E_n = 0$. Choose $w \in E_n$ lifting $\bar{v}_0$ in the quotient of (3.2). The vectors $w, fw, \dots, f^n w$ map to a basis of that quotient. They span a U-submodule, since

$$hf^i w = (n-2i)f^i w, \qquad ef^i w = i(n-i+1)f^{i-1}w, \qquad f^{n+1}w = 0.$$

Here the first formula follows from $hw = nw$ and $hf = f(h-2)$, the second from (3.4) with $r = i$ together with $ew = 0$ and $hw = nw$, and the third from (3.3), since $w \in E_n$. This submodule maps isomorphically to V_n, giving the desired splitting. □

The following theorem is Theorem A for the algebra of Construction 2.1 and the modules of Construction 3.1. It also records the dimension vectors of these modules.

Theorem 3.3. *The algebra Λ of Construction 2.1 is connected and basic, has dimension 21 and global dimension 2, and satisfies $(\operatorname{rad}\Lambda)^3 = 0$. The modules of Construction 3.1 have dimension vectors $\underline{\dim} B_n = (n+1, n+1, n+1)$ and satisfy*

$$\operatorname{Hom}_\Lambda(B_m, B_n) \cong \begin{cases} k & m = n, \\ 0 & m \neq n, \end{cases} \qquad \operatorname{Ext}^1_\Lambda(B_m, B_n) = 0 \quad (m, n \geq 0).$$

Consequently, $\{B_n \mid n \geq 0\}$ is an infinite semibrick of pairwise extension-orthogonal rigid bricks.

Proof. The assertions about Λ follow from Proposition 2.2. Construction 2.3 gives the dimension vectors. The Hom and Ext assertions follow from (2.4) and Lemma 3.2. □

Finally, we recall the argument from [3, Section 4] mentioned in the introduction.

Remark 3.4. Let Γ be a finite-dimensional algebra with Auslander–Reiten translation τ. Let X be a Γ-module of projective dimension at most one. Then for every Γ-module Y, we have $\operatorname{Ext}^1_\Gamma(X, Y) = 0$ if and only if $\operatorname{Hom}_\Gamma(Y, \tau X) = 0$ [2, Corollary IV.4.7]. Hence, if finitely many pairwise extension-orthogonal rigid bricks have projective dimension at most one, their direct sum M satisfies $\operatorname{Hom}_\Gamma(M, \tau M) = 0$, that is, M is τ-rigid. A τ-rigid module has at most as many pairwise nonisomorphic indecomposable direct summands as Γ has simple modules [1, Proposition 1.3]. Therefore only finitely many members of such a semibrick have projective dimension at most one. Hence, if Γ has an infinite semibrick of pairwise extension-orthogonal rigid bricks, then Γ has global dimension at least two.

For our modules, the next proposition shows that every B_n has projective dimension two. It also computes the dimensions of the second extensions from the resolutions (2.3).

Proposition 3.5. *For all $m, n \geq 0$,*

$$\dim_k \operatorname{Ext}^2_\Lambda(B_m, B_n) = (m+1)(n+1) - \delta_{mn},$$

where δ_{mn} is 1 if $m = n$ and 0 otherwise. Every B_m has projective dimension 2.

Proof. For Λ-modules X and Y with dimension vectors $d = (d_1, d_2, d_3)$ and $u = (u_1, u_2, u_3)$, respectively, put $\chi(X, Y) = \sum_{i=0}^{2} (-1)^i \dim_k \operatorname{Ext}^i_\Lambda(X, Y)$. Since Λ has global dimension 2, the long exact sequence shows that χ is additive on short exact sequences in X. Hence $\chi(X, Y) = \sum_{j=1}^{3} d_j \, \chi(S_j, Y)$. By (2.3) and $\operatorname{Hom}_\Lambda(P_i, Y) \cong Y_i$,

$$\chi(S_1, Y) = u_1 - 4u_2 + 6u_3, \qquad \chi(S_2, Y) = u_2 - 4u_3, \qquad \chi(S_3, Y) = u_3,$$

and therefore

$$\chi(X, Y) = d_1 u_1 + (d_2 - 4d_1)u_2 + (d_3 - 4d_2 + 6d_1)u_3.$$

Hence $\chi(B_m, B_n) = (m+1)(n+1)(1 - 3 + 3) = (m+1)(n+1)$. Since Theorem 3.3 gives

$$\dim_k \operatorname{Hom}_\Lambda(B_m, B_n) = \delta_{mn}, \qquad \operatorname{Ext}^1_\Lambda(B_m, B_n) = 0,$$

the dimension of $\operatorname{Ext}^2_\Lambda(B_m, B_n)$ is $(m+1)(n+1) - \delta_{mn}$. For each m, choose $n \neq m$. Then $\operatorname{Ext}^2_\Lambda(B_m, B_n) \neq 0$, so B_m has projective dimension 2. $\square$

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